Adaptive Learning, Heterogeneous Expectations and Forward Guidance

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Abstract

In a Cagan-type model of inflation we investigate how announcements of forward guidance may help lower inflation persistence. An important component of the model is heterogenous expectation formation: while market participants consider a time series model that uses sample autocorrelation learning, the Central Bank uses least squares learning of the MSV solution. Under certain parameter settings a high and low persistence equilibrium exist. We find that while active monetary policy does reduce the amount of time spent near the high persistence equilibrium, it is not enough to ensure that only the low persistence equilibrium obtains. Forward guidance in the form of announcements of future economic conditions, on the other hand, eliminates the possibility of a high inflation equilibrium. The intuition behind this result is that forward guidance can “shock” agents expectations in a way that overcomes the pull of persistence.

Keywords: adaptive learning, forward guidance, heterogeneous expectations.

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1 Introduction

As noted by Ascari and Sbordone (2014), in practice price stability usually means a moderate rate of inflation. Monetary policy makers are content with some inflation, but not too much. Macroeconomists have noted that the main problem in taking macroeconomic models to the data, is accounting for inflation persistence (see Fuhrer and Moore (1995)). Inflation persistence can lead to trends in inflation that are higher than the moderate rate with which policy makers would be comfortable (for example, the high inflation rates in the US during the 70s and 80s). The solution to this problem has been to target a particular inflation rate by monitoring and controlling interest rates. Levin, Natalucci and Piger (2004) provide empirical evidence that inflation targeting results in lower inflation persistence. Recently, however, monetary policy makers have reduced interest rates to the lower bound of zero, limiting policy makers ability to use their most powerful tool in response to inflation rates below target levels. The means by which policy makers might influence inflation rates at the lower bound is still an open question. In particular one might be concerned about the effectiveness of such unconventional policy.

Having lowered interest rates to the lower bound, central bankers have relied on what they call “forward guidance” as one unconventional tool to manage inflation expectations and market participant’s economic outlook thus maintaining the target inflation rate. The most common way to think of forward guidance is a commitment by the central bank to maintain a low interest rate (or not raise rates) for a specified period of time. Monetary policy makers are faced with a conundrum since forward guidance may elicit two responses. Some agents respond to the anticipated real effects of the accommodative policy, while others respond to the announcement that the economy requires a significant amount of time to recover. This paper specifically focuses on the latter. This work can be applied more broadly to any central bank communication that informs the public about central bank expectations of future economic conditions.
Central bank communication has largely been defined and modeled as providing the monetary policy rule that central bankers use. In fact, a large portion of the literature investigates how increased transparency can help steer the economy toward a desirable path. Rudebusch and Williams (2008) argue that increased transparency, in the form of explicit interest rate forecasts, can lead to a better ability by the central bank to achieve its’ goals. This theoretical work helps explain empirical results found by Swanson (2006). While we only provide highlights on central bank communication here, the interested reader should see Blinder, Ehrmann, Fratzscher, de Haan and Jansen (2008) for a more general overview on the subject. In contrast to communication, forward guidance provides market participants with information about where the central bank believes the economy is headed in the long run. In some sense forward guidance provides less information, but the benefit is flexibility, particularly at the lower bound. Central bank communication in contrast is typically thought of as providing a rule to justify the adjustment of interest rates, which is useless at the zero lower bound, except, perhaps, for assessing the magnitude of rate increases once lift off from the lower bound occurs. Instead of providing a policy rule explicitly, a key point of forward guidance relies on agents adjusting their expectations. Consequently this paper emphasizes this channel of influence. Our argument is that market participants are less equipped to forecast the severity or duration of economic cycles, and therefore central banks can provide critical information that may help stabilize an economy.

While policy makers have been actively using forward guidance, the literature on the topic generates mixed results under rational expectations. Empirical evidence suggests that unconventional policy, which includes quantitative easing in addition to forward guidance, has similar effects as conventional on output while incurring minimal impact on prices (Gambacorta, Hofmann and Peersman 2014). On the other hand, Del Negro, Giannoni and Patterson (2012) show that medium scale DSGE models with rational expectations overstate the impact of forward guidance. Kulish, Morley and Robinson (2014) corroborate this conclusion, finding that the average expected duration of the
zero lower bound regime following the financial crisis was 3 quarters. While the estimates of expected duration vary significantly, we have been at the zero lower bound since the first quarter of 2009, which suggests the rational expectations DSGE model drastically overstates the impact of forward guidance. Recent theoretical work by Magill and Quinzii (2013) show that under rational expectations forward guidance can lead to determinacy. In particular, they show that equilibria are indeterminant if policy makers only use a rule to fix short-term interest rates. However, as demonstrated by Gaballo (2013) the relationship between short-term policy and forward guidance may not always work perfectly if agents are rationally inattentive. Specifically, risks, both individual and aggregate, increase when policy is loose and agents are relatively inattentive. On some level these mixed results should not be surprising since forward guidance is an attempt to influence expectations to affect the economy, or put the expectations/real effects in disequilibrium, which is exactly not the type of effects the rational expectations paradigm is capable of explaining.

As pointed out by Bernanke (2004), monetary policy should be robust to non-rational expectations and given these tenuous results under rational expectations, one might be concerned that forward guidance may not be effective in a model where the rational expectations assumption has been relaxed. In addition, empirical evidence from Lansing (2009) suggests that expectations may play a large role in the time varying dynamics of inflation. While previous theoretical research has investigated central bank communication under adaptive learning, see Preston and Eusepi (2010), few models of forward guidance and learning exist. Honkapohja and Mitra (2014) examine the robustness of three specific policy frameworks under adaptive learning and forward guidance. They find that nominal GDP and price level targeting results under learning are not robust (compared to rational expectations) unless guidance is provided.

This paper attempts to add to this literature by focusing on the impact of information contained in a forward guidance announcement on inflation persistence.
key factor in creating this effect in the model described by this paper is allowing the central bank first mover advantage when forming expectations and then announcing their perceived state of the economy. Market participants consequently incorporate the central bank announcements concerning potential future economics conditions into their expectation formation process. Both agents use simple, but different, econometric models to predict inflation, and have access to slightly different information sets. In particular, market participants believe inflation is autoregressive and stationary. Consequently they estimate the relationship using sample autocorrelation (SAC). Central bankers on the other hand have an accurate perception of the reduced form model of the economy under rational expectations, that is output can influence inflation. They assume rational expectations because they do not know how private agents form expectations. Policy makers use this model to predict inflation for the coming period and enact policy accordingly. Under forward guidance the central bank uses forward looking expectations to predict the future state of the economy in the subsequent period. The policy actions are purposefully simplified in this model so that policy makers can directly influence inflation as in Sargent, Williams and Zha (2009). This simplification allows us to focus on the immediate results of forward guidance and the mechanism by which the results are obtained.

The story here is that market participants, left to their own devices, might lead the economy into a bad equilibrium, but central bank announcements can help guide market participants expectations toward a more desirable result. One should note, however, that all of the results are obtained in a model without interest rates as a direct policy tool (that is no interest rate rule). This is not necessarily a drawback since interest rates are controlled by central bankers by influencing the money supply, which is the monetary policy mechanism our model uses. This choice also allows for the opportunity to focus specifically on the announcement channel of forward guidance.

Another important aspect of the model is the existence of multiple equilibria, one with low autocorrelation and one with high autocorrelation. As seen in Figure 1
which displays monthly CPI inflation, and 4 year rolling window of autocorrelation and average inflation, there is some rough empirical evidence for multiple equilibria. While it is unclear whether structural breaks exist, it is clear that there are periods of high and low persistence. Our model is designed to generate this stylized fact. In the model we find that when the central bank is not conducting active monetary policy it is possible for the economy to switch between the high persistence and low persistence equilibria. However, when policy makers engage in active policy they can reduce the amount of time spent near the high persistence equilibrium. Furthermore, providing forward guidance eliminates the possibility of temporarily settling on the high persistence equilibrium.

The next section describes the modeling framework, the rational expectations so-
lution, and the Stochastic Consistent Expectations Equilibria (SCEE) that arise from SAC learning. Section three introduces central bank policy and its influence in reducing inflation persistence. The subsequent section describes the forward guidance mechanism and the results of its influence on autocorrelation. Section five presents parameter sensitivity results and the last section concludes.

2 Model

We use a model similar to the seminal work of Cagan (1956) on hyperinflation. While slightly simplistic from the point of view of monetary policy it will allow us to focus on the expectations channel. In the model real money demand is a log-linear function of real output and the nominal interest rate:

\[
\log \left( \frac{M^d_t}{P_t} \right) = \lambda \log (Y_t) - \beta i_t + u_t
\]

where, \(P_t\) is the price level at time \(t\), \(\lambda > 0\), \(\beta > 0\), \(Y_t\) is real output, \(i_t\) is the nominal interest rate, and \(u_t\) is a Gaussian error term with variance \(\sigma_u^2\). Money supply is determined by the monetary authority in the following way:

\[
M^s_t = \omega P_{t-1}
\]

In baseline model we keep the policy parameter \(\omega\) fixed and refer to this as passive monetary policy. This is similar to Evans and Ramey (2006) which allows the monetary policy makers to adjust to the previous periods price level. When we incorporate active monetary policy in the subsequent section we will be assuming that policy makers adjust \(\omega\) depending on their inflation targets.

Solving for equilibrium and using lower case letters to represent the logarithm of
the appropriate variable we have:

\[ \log(\omega) + p_{t-1} - pt = \lambda y_t - \beta i_t + u_t \]  

(3)

Since our intent is to describe the ability of monetary policy and forward guidance to influence inflation not output we assume that \( y_t \) is exogenously determined by an autoregressive process. Defining inflation as \( \pi_t = pt - p_{t-1} \) one can rewrite equation (3) as \( i_t = \beta^{-1}(\pi_t - \log(\omega) + \lambda y_t + u_t) \), which looks very similar to a Taylor-type monetary policy rule. Thus, while monetary policy is not defined as an interest rate rule, it has an impact on nominal interest rates. Further if we define the nominal interest rate as:

\[ i_t = r + E_{t-1} \pi_{t+1} \], where the real interest rate, \( r \), is held constant and expectations are formed by market participants forecasts we have:

\[ \pi_t = \alpha + \beta E_{t-1} \pi_{t+1} - \lambda y_t - u_t \]  

(4)

\[ y_t = \rho y_{t-1} + g_t \]  

(5)

where \( \alpha = \log(\omega) + \beta r \), \(-1 < \rho < 1 \), and \( g_t \) is a Gaussian error with variance \( \sigma^2_g \). Note that given a constant real interest rate \( \alpha \) is directly influenced by the policy parameter. Further note that set up of the model does not allow for zero lower bound considerations.\(^\text{1}\)

First we describe the time series properties of the rational expectations solution of this model. The minimum state variable (MSV) solution of the following form

\[ \pi_t = b_0 + b_1 y_{t-1} + u_t \]  

(6)

Using (6) to form agents’ expectations for time \( t + 1 \) given \( t - 1 \) information results in rational expectations solution of \( b_{0}^{RE} = \alpha/(1 - \beta) \) and \( b_{1}^{RE} = -\lambda \rho/(1 - \beta \rho) \). Since market participants form expectations based on sample autocorrections we calculate

\(^{1}\)For more on the limitations of forward guidance at the lower bound see Levin, López-Salido, Nelson and Yun (2010).
the autocorrelation of the rational expectations solution for comparison:

$$\text{Corr}(\pi_t, \pi_{t-1}) = \frac{\lambda^2 \rho (\rho + (1 - \beta \rho)(1 - \rho^2))}{\lambda^2 \rho^2 + (1 - \beta \rho)(1 - \rho^2)(\lambda^2 + \frac{\sigma_y^2}{\sigma_y^2})}$$ \hspace{1cm} (7)

To familiarize the reader the stability concept used in adaptive learning we examine the model if agents use least squares to estimate $b_0$ and $b_1$. The stability concept is called Expectational-Stability or E-stability, which maintains that if, under a suitable econometric specification, agents’ coefficient estimates asymptotically converge to a particular equilibrium (in this case the rational expectations solution) with probability one then the equilibrium is E-stable. The preferred method for determining E-stability relies on calculating the eigenvalues of the derivative of the ordinary differential equations determined by the model and the recursive algorithm agents use to update their estimates. The recursive form of least squares is

$$\hat{\phi}_{i,t} = \hat{\phi}_{i,t-1} + \gamma_t R^{-1}_t z_{t-1} (\pi_{i,t} - \hat{\phi}'_{i,t-1} z_{t-1})$$

$$R_t = R_{t-1} + \gamma_t (z_{t-1} z'_{t-1} - R_{t-1})$$

where $\hat{\phi}_t = (\hat{b}_0, \hat{b}_1)'$, $z_t = (1, y_t)'$, $R_t$ is the moment matrix or, with a slight abuse of notation, the $X'X$ of a least squares estimator, and $\gamma_t$ is the gain parameter. For asymptotic purposes one can set $\gamma_t = 1/t$, which is called decreasing gain learning.

A convenient way to determine the E-stability condition is to compare the agents perceived law of motion (PLM), equation (8), to the actual law of motion (ALM). The ALM is found by substituting the PLM into (4), which results in:

$$\pi_t = \alpha + \beta b_0 + (\beta b_1 - \lambda) \rho y_{t-1} - u_t \hspace{1cm} (8)$$

There exists a mapping between the PLM and ALM referred to as the T-map $(\hat{b}_0, \hat{b}_1) = (\alpha + \beta b_0, \beta b_1 - \lambda)$. If the eigenvalues of the derivative of this system of equations have real parts less than one then the model is E-stable. For this particular model the
system will converge to the rational expectations solution if $\beta < 1$.

### 2.1 SAC Learning

Now we consider this model assuming a representative agent, standing in for market participants, that uses SAC learning and passive monetary policy. Agents’ PLM is the following:

$$\pi_t = a_0 + a_1(\pi_{t-1} - a_0) + e_t$$  \hspace{1cm} (9)

where $a_0$ is the mean of $\pi_t$ and $a_1$ is the first order autocorrelation coefficient. Substituting in the PLM in to (4) we find the ALM,

$$\pi_t = \alpha + \beta(a_0 + a_1^2(\pi_{t-1} - a_0)) - \lambda y_t - u_t.$$  \hspace{1cm} (10)

The equilibrium concept used for this type of learning is called Stochastic Consistent Expectations Equilibrium (See (Hommes and Sorger 1998)). This concept ensures that the underlying data (in this case inflation) confirms agents’ beliefs of autocorrelation, that is, agents perceptions match reality. It follows that the unconditional mean of inflation, $\hat{\pi}$, is

$$\hat{\pi} = \frac{\alpha + \beta a_0(1 - a_1^2)}{1 - \beta a_1^2}$$  \hspace{1cm} (11)

To see this, recognize that the unconditional means of $x$ and $u$ are both zero and set $\pi_t = \pi_{t-1} = \hat{\pi}$. To be consistent, agents’ estimate of the mean, $a_0$, should equal the theoretical mean of inflation, $\hat{\pi}$. Solving for $a_0$ yields our SCEE mean $a_0^* = \alpha/(1 - \beta)$, which also happens to be the mean under rational expectations found above. In addition, $a_1$ is equal to the first order autocorrelation coefficient of the ALM.

$$\text{Corr}(\pi_t,\pi_{t-1}) = \beta a_1^2 + \frac{\lambda^2 \rho(1 - \beta^2 a_1^4)}{\lambda^2(\beta \rho a_1^2 + 1) + (1 - \rho^2)(1 - \beta \rho a_1^2) \frac{\sigma_x^2}{\sigma_u^2}}$$  \hspace{1cm} (12)
As shown in Hommes and Zhu (2014) there may be as many as 3 solutions for $a_1$ based on this equation. Figure 2 demonstrates the three possible equilibria.

In the context of learning we are interested in which of these equilibria are E-stable. Graphically Figure 2 shows that the lowest and highest equilibria are E-stable. With these parameter values the two E-stable equilibria are $a_1^* \approx 0.3807$ and $a_1^* \approx 0.9969$ and the one E-unstable equilibria when $a_1^* \approx 0.6599$. For comparison the autocorrelation of the RE solution, found in equation (7), for these parameter values is approximately 0.7179.

While convergence is a nice property for theoretical models, but in the presence of multiple equilibria it makes sense to assume that agents may be wary of structural
Figure 3: Switching Between Equilibria

Note: The first row depicts realized inflation rates, the second, agents autocorrelation estimates, and the last agents mean estimates.

changes. A common assumption to handle structural change is to set the gain parameter to some constant, $\gamma_t = \gamma$. The recursive algorithm for updating agents sample autocorrelation coefficients can be found in the online appendix to Hommes and Zhu (2014), which we replicate here for the readers convenience.

$$a_{0,t} = a_{0,t-1} + \gamma(y_t - a_{0,t-1})$$

$$a_{1,t} = a_{1,t-1} + \gamma R_t^{-1}((y_t - a_{0,t-1})(y_{t-1} - a_{0,t-1}) - a_{1,t-1}(y_t - a_{0,t-1})^2)$$

$$R_t = R_{t-1} - \gamma((y_t - a_{0,t-1})^2 - R_{t-1})$$

Figure 3 demonstrates the effect of assuming a constant gain of $\gamma = 0.008$. The main
conclusion is that it is possible to switch between the two stable equilibria. This point is made in the online appendix of Hommes and Zhu (2014), but is necessary for comparisons made later in the paper. Note how qualitatively similar these graphs are to real data of Figure 1. Other simulations yield similar results, but the timing and magnitude of this particular series will be instructive for the results presented below.

To assess the effectiveness of passive monetary policy we calculate frequency with which the economy is near the high autocorrelation equilibrium. We do this by calculating the percentage of periods spent above the unstable equilibria (0.7411). Calculations begin after 20,000 (of 40,000) periods for 1,000 simulations. The average percentage spent above the unstable equilibria was 30.99 percent. Presumably a policy maker would want to avoid or reduce the time spent in the high persistence equilibrium in favor of the low persistence one. The next two sections describe active monetary policy and forward guidance and demonstrate how both influence the likelihood of remaining in the low persistence equilibrium.

3 Monetary Policy Results

This section describes the stylized monetary policy mechanism. Our assumption is that monetary policy makers are more sophisticated than our market participants. That is, they have some knowledge of how the economy may work. In particular, we assume that they understand that output influences inflation, or that their PLM is the same as the MSV solution in equation (6) above. Another way of looking at this assumption is that both market participants and policy makers have consistent long run models (the equilibrium means are identical), however, perceptions of the short run dynamics differ. The central bank will also be concerned about structural breaks and will therefore use a constant gain.

Active monetary policy is defined here as policy makers adjusting their policy parameter $\omega$ to respond more aggressively to prices. When they believe inflation is too far
below (above) their target they increase (decrease) the policy parameter. Recall that under our assumptions, $\alpha$ is directly affected by $\omega$. We calibrate the monetary policy reaction to increase or decrease the long run (or the rational expectations solution) inflation rate by one half of a percent.

To ensure that the policy announcements are linked with the idea of forward guidance we assume that the policy makers consider multiple periods. Since policy makers do not want to respond to shocks they use a nowcast (the expectation of inflation in the current period) and inflation in the previous period. If they expect to see two periods of inflation too far below (above) their target the attempt to inflate (deflate) the economy.

\[
\alpha_t = \begin{cases} 
\alpha_h & \text{if } \pi_{t-1} < \bar{\alpha}_l/(1 - \beta) \text{ and } \pi_{t}^{CB,e} < \bar{\alpha}_l/(1 - \beta), \\
\alpha_l & \text{if } \pi_{t-1} > \bar{\alpha}_h/(1 - \beta) \text{ and } \pi_{t}^{CB,e} > \bar{\alpha}_h/(1 - \beta), \\
\bar{\alpha} & \text{otherwise.}
\end{cases}
\] (13)

For simplicity and symmetry we set $\bar{\alpha}_l = \alpha_l$ and $\bar{\alpha}_h = \alpha_h$. Since the policy is inherently countercyclical, it drives the inflation process in the direction of the unconditional mean. Recall also that the equilibrium sample autocorrelation coefficient ($a_{1t}^*$) is not a function of $\alpha$ and therefore will not be directly influenced.

Since an analytical solution for the equilibria is rather difficult to obtain under active monetary policy, we use the asymptotic properties of learning to confirm the existence of two stable equilibria. We run 2 sets 1,000 simulations each of 50,000 periods in length under decreasing gain learning. The first set starts near the high equilibria, and the second near the low equilibria. At the end of each simulation we collect the average of the estimates autocorrelation, $a_t$, over the last 5,000 periods and take the average for each set. We find the high equilibrium is approximately around 0.9935

\footnote{As long as the upper and lower bounds do bind on occasion the results are not that sensitive to this assumption. For the simulations we set $\alpha_l = 0.000015$ and $\alpha_h = 0.000025$ following the policy calibration discussion above.}
Figure 4: Effects of Active Monetary Policy

Note: The first row depicts realized inflation rates, the second, agents autocorrelation estimates, and the last agents mean estimates.

with a standard deviation of 0.0012. The low equilibrium occurs around 0.3362 with a standard deviation of 0.0186. Statistically these are both different, and slightly lower than the analytical results found in the previous section, however, they both exist, and are not wildly different from the passive monetary policy case.

Figure 4 uses the same series of shocks that created Figure 3, but with the monetary policy regimes dictated by \[13\]. Notice that although the general series is qualitatively similar the inflation rate is not nearly as high during the “high inflation” period. We also can observe that the amount of time spent near the high autocorrelation solution has been reduced quite dramatically. These results suggest that monetary policy of
this form can help reduce the amount of inflation possible if the economy has potential to converge on a bad equilibrium. Using the same procedure as in the previous section, we find that the economy is near the high persistence equilibrium for only 14.01 percent of the periods. The question then becomes whether forward guidance can approach this level of success.

4 Forward Guidance Results

Forward guidance can typically be thought of as a commitment by the central bank to maintain low interest rates (or not raise rates) for a specified period of time. The announcement of policy makers intentions signals two thing to market participants, first borrowing conditions will remain the same (and favorable?) for a defined period of time and second, the central bank expects weak economic growth and little pressure on inflation. The first signal influences agents decision making on a micro level, that this model will not be able to address. However, the second signal is exactly what our model attempts to capture. The basic narrative is that the central bank makes an announcement that provides the market participant with a signal of future economic conditions. Market participants then adjust their expectations accordingly. For example, an announcement that the Federal Reserve plans expects to raise interest rates by the middle of 2015 would indicate that economic conditions are improving and market participants might expect inflation rates closer to normal times.

For consistency, the central bank announcements follow the same basic functional form as active monetary policy. Therefore we assume that monetary authority just reports the expected state of the economy in the next period. That is, whether they
expect low, high, or just status quo levels of inflation in the following periods.

\[ A_t = \begin{cases} 
1 & \text{if } \pi^C_{t} < \bar{\alpha}_l/(1 - \beta) \text{ and } \pi^C_{t+1} < \bar{\alpha}_l/(1 - \beta), \\
2 & \text{if } \pi^C_{t} > \bar{\alpha}_h/(1 - \beta) \text{ and } \pi^C_{t+1} > \bar{\alpha}_h/(1 - \beta), \\
3 & \text{otherwise.} 
\] (14)

Alternatively, policy makers could provide their expected inflation rates as a part of market participants information set. This is unlikely to affect the results. If instead policy makers were engaging in active monetary policy one could assume the announcement is of expected future policy. We stick with the more simplistic assumption so that policy announcements, \( A_t \), observe a similar functional form as policy. The only difference is that the central bank uses expectations of today and the next period to forecast the expected state of the world.

Incorporating announcements in agents expectations might also be modeled several ways. We choose think of our agents acting as if the understand the idea of a regime switching model. That is, agents are aware that there may exist a different long term inflation rate during each of the three different economic conditions. Therefore, market participants keep track of three intercepts, \( a^i_{t,t} \) for \( i = 1, 2, 3 \), associated with each of the announcements. Each period they only update the mean associated with the announcement made by the central bank. The other means remain the same. The following set of equations demonstrates how the standard SAC learning is adjusted to

\[ a^i_{t,t} = \begin{cases} 
1 & \text{if } \pi^C_{t} < \bar{\alpha}_l/(1 - \beta) \text{ and } \pi^C_{t+1} < \bar{\alpha}_l/(1 - \beta), \\
2 & \text{if } \pi^C_{t} > \bar{\alpha}_h/(1 - \beta) \text{ and } \pi^C_{t+1} > \bar{\alpha}_h/(1 - \beta), \\
3 & \text{otherwise.} 
\]
Note: The first row depicts realized inflation rates, the second, agents autocorrelation estimates, and the last agents mean estimates.

incorporate these assumptions.

\[
a_{0,t}^i = \begin{cases} 
    a_{0,t-1}^i + \gamma (y_t - a_{0,t-1}^i) & \text{if } A_t = i \\
    a_{0,t-1}^i & \text{otherwise}
\end{cases} \quad \text{(15)}
\]

\[
a_{0,t} = a_{0,t}^{A_t} \quad \text{(16)}
\]

\[
R_t = R_{t-1} - \gamma ((y_t - a_{0,t-1}^i)^2 - R_{t-1}) \quad \text{(17)}
\]

\[
a_{1,t} = a_{1,t-1} + \gamma R_{t-1}^{-1}((y_t - a_{0,t-1}^i)(y_{t-1} - a_{1,t-1}) - a_{0,t-1}^i(y_t - a_{0,t-1}^i)^2) \quad \text{(18)}
\]

Effectively this makes our market participants more sophisticated econometrically, be-
cause they are now, in some sense, estimating a regime switching model. However they rely on the announcements provided by the monetary authority to determine when the switches occur. The other feature of this assumption is that credibility will be straightforward to identify. If the central bank announcements are not credible then there should not be a significant difference between the means of each state of the economy.

Figure 5 displays the results from assuming central bankers only provide forward guidance and conduct no active monetary policy ($\alpha$ is constant). This delivers striking result that the period of high autocorrelation in the middle of the sample has been completely eliminated. Following the same procedure as the previous sections, we find no instances over 1000 simulations of persistence approaching the high equilibrium levels. The intuition is straightforward; monetary policy makers provide sufficient accurate information about the future such that shifts in agents expectations undercut the persistence. Accuracy in this case can be approximated by noting that $a_0^1$, $a_0^2$, and $a_0^3$ are appropriately ordered. At the end of the benchmark simulation in Figure 5 we find that $a_0^1 = 0.0207$, $a_0^2 = 0.0258$, and $a_0^3 = 0.0233$ corresponding to the low inflation, high inflation, and target inflation states.

In addition, notice that the agents estimates of autocorrelation are much lower than the low persistence equilibrium mentioned above. Rerunning the simulation under a decreasing gain and taking the average of the last 5000 values for 1,000 simulations yields a average value of 0.1206 with an average standard deviation of 0.0366. Under forward guidance the high persistence equilibrium is eliminated and the degree of autocorrelation in other equilibrium is decreased. The mechanism by which this occurs is through central bank announcements influencing market participants expectations. Figure 6 illustrates this by focusing on the 4000 period episode of high inflation in the benchmark simulation. Monetary policy alone only reduces the persistence of market participants expectations thereby lowering persistence of the resulting inflation. Market participant expectations under forward guidance are strikingly different. The
Figure 6: Effects of Forward Guidance

Note: Row 1 is the baseline with no monetary policy or forward guidance. Rows 2 and 3 are market participants and central bank expectations (respectively) with active monetary policy. Rows 4 and 5 are market participants and central bank expectations (respectively) with forward guidance.

Abrupt changes in expectation due to forward guidance drives down persistence and acts as a moderating influence on the resulting inflation. Note that with active monetary policy central bank and market participant expectations are quite similar, but that is not the case under forward guidance.

5 Parameter Sensitivity

A typical concern with numerical analysis is sensitivity to parameter choice. Some of this is mitigated by using values with precedent, but we attempt to illustrate the robustness of the results by varying the parameters around the values that generate
Table 1: Parameter Sensitivity Results: $\beta$

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<thead>
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<th>Stable Equilibria</th>
<th>Autocorrelation</th>
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</tr>
<tr>
<td>0.950</td>
<td>0.358</td>
<td>0.970</td>
</tr>
<tr>
<td>0.955</td>
<td>0.361</td>
<td>0.975</td>
</tr>
<tr>
<td>0.960</td>
<td>0.363</td>
<td>0.980</td>
</tr>
<tr>
<td>0.965</td>
<td>0.366</td>
<td>0.984</td>
</tr>
<tr>
<td>0.970</td>
<td>0.369</td>
<td>0.987</td>
</tr>
<tr>
<td>0.975</td>
<td>0.372</td>
<td>0.990</td>
</tr>
<tr>
<td>0.980</td>
<td>0.375</td>
<td>0.993</td>
</tr>
<tr>
<td>0.985</td>
<td>0.378</td>
<td>0.995</td>
</tr>
<tr>
<td>0.990</td>
<td>0.381</td>
<td>0.997</td>
</tr>
<tr>
<td>0.995</td>
<td>0.385</td>
<td>0.998</td>
</tr>
</tbody>
</table>

$\rho = 0.9$ and $\lambda = 0.8$

The results above. The main contention of the paper is that forward guidance may lead to lower overall autocorrelation and therefore monetary policy makers will be more likely to achieve their inflation target. In varying the parameters we find that there is a discontinuity when the underlying autocorrelation in output ($\rho$) is small enough, and also when the relationship between output and inflation ($\lambda$) is weak. This discontinuity is likely due to the policy parameters (specifically the cutoffs for the announcements) not being chosen optimally.

Table 1 displays there results when varying the discount factor, $\beta$. We find that below 0.930 there exists only one $E$-stable equilibrium. In addition for all the values the mean (standard deviation) autocorrelation of the last 5000 periods for each of the 1000 simulations. The standard deviations imply that the autocorrelations with forward guidance are not statistically different from each other, but the trend of the mean is
Table 2: Parameter Sensitivity Results: $\rho$

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>Stable Equilibria</th>
<th>Autocorrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>0.800</td>
<td>0.142</td>
<td>-</td>
</tr>
<tr>
<td>0.810</td>
<td>0.151</td>
<td>-</td>
</tr>
<tr>
<td>0.820</td>
<td>0.162</td>
<td>-</td>
</tr>
<tr>
<td>0.830</td>
<td>0.174</td>
<td>0.968</td>
</tr>
<tr>
<td>0.840</td>
<td>0.188</td>
<td>0.983</td>
</tr>
<tr>
<td>0.850</td>
<td>0.204</td>
<td>0.988</td>
</tr>
<tr>
<td>0.860</td>
<td>0.223</td>
<td>0.991</td>
</tr>
<tr>
<td>0.870</td>
<td>0.247</td>
<td>0.994</td>
</tr>
<tr>
<td>0.880</td>
<td>0.277</td>
<td>0.995</td>
</tr>
<tr>
<td>0.890</td>
<td>0.318</td>
<td>0.996</td>
</tr>
<tr>
<td>0.900</td>
<td>0.381</td>
<td>0.997</td>
</tr>
<tr>
<td>0.910</td>
<td>-</td>
<td>0.997</td>
</tr>
<tr>
<td>0.920</td>
<td>-</td>
<td>0.998</td>
</tr>
<tr>
<td>0.930</td>
<td>-</td>
<td>1.000</td>
</tr>
<tr>
<td>0.940</td>
<td>-</td>
<td>0.999</td>
</tr>
</tbody>
</table>

$\beta = 0.99$ and $\lambda = 0.8$

fairly clear; as the value of $\beta$ decreases autocorrelation decreases as well.

However, when we turn to underlying autocorrelation of the output process ($\rho$) we can observe, on Table (2), the transition from one low equilibrium, $\rho < 0.83$, to two stable equilibria, $0.83 \leq \rho < 0.91$, and then back to one high equilibrium, $\rho > 0.91$. In addition the story about the autocorrelation under forward guidance is a little more complicated. When $\rho > 0.86$ the autocorrelation under forward guidance is lower, but at or below 0.86 the autocorrelation value starts to increase. Taken in context with Table (3), where we see the same discontinuity, this suggests some interplay between these two parameters. For the slope of the relationship between output and inflation ($\lambda$) the beneficial effects of forward guidance disappear when the parameter value falls below 0.066. This result is due to the upper and lower bounds remaining fixed regardless of the underlying structural parameters. Since $\lambda$ and $\rho$ influence the magnitude of the fluctuations of inflation as they decrease the monetary policy cutoffs are no longer binding which leads the actual autocorrelation to rise. The actual autocorrelation values calculated from simulations are different than the theoretical values because in
Table 3: Parameter Sensitivity Results: $\lambda$

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>Stable Equilibria</th>
<th>Autocorrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>0.060</td>
<td>0.177</td>
<td>0.993</td>
</tr>
<tr>
<td>0.062</td>
<td>0.191</td>
<td>0.994</td>
</tr>
<tr>
<td>0.064</td>
<td>0.205</td>
<td>0.994</td>
</tr>
<tr>
<td>0.066</td>
<td>0.220</td>
<td>0.995</td>
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<tr>
<td>0.068</td>
<td>0.237</td>
<td>0.995</td>
</tr>
<tr>
<td>0.070</td>
<td>0.254</td>
<td>0.996</td>
</tr>
<tr>
<td>0.072</td>
<td>0.274</td>
<td>0.996</td>
</tr>
<tr>
<td>0.074</td>
<td>0.295</td>
<td>0.996</td>
</tr>
<tr>
<td>0.076</td>
<td>0.319</td>
<td>0.996</td>
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<tr>
<td>0.078</td>
<td>0.347</td>
<td>0.997</td>
</tr>
<tr>
<td>0.080</td>
<td>0.381</td>
<td>0.997</td>
</tr>
<tr>
<td>0.082</td>
<td>0.428</td>
<td>0.997</td>
</tr>
<tr>
<td>0.084</td>
<td>-</td>
<td>0.997</td>
</tr>
<tr>
<td>0.086</td>
<td>-</td>
<td>0.997</td>
</tr>
<tr>
<td>0.088</td>
<td>-</td>
<td>0.997</td>
</tr>
<tr>
<td>0.090</td>
<td>-</td>
<td>0.997</td>
</tr>
</tbody>
</table>

$\rho = 0.9$ and $\beta = 0.99$

the simulations forward guidance lowers the persistence, whereas the theoretical values are based on passive monetary policy.

6 Conclusion

The model considered in this paper displays multiple equilibria that can lead to ineffective monetary policy. The two stable equilibria, a result of SAC learning, can be categorized as a high and low autocorrelation states. In the high autocorrelation state central bank targets are not always effective at maintaining the desired inflation rate. However, when policy makers engage in forward guidance the low autocorrelation equilibria is lower and the only equilibrium obtained. This suggests that forward guidance can have an important impact on maintaining central bank inflation targets. Further research should investigate whether these results would hold in a model which accounted for the zero lower bound on interest rates.
References


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Swanson, Eric, “Have Increases in Federal Reserve Transparency Improved Private Sector Interest Rate Forecasts?,” Journal of Money, Credit and Banking, April 2006, 38 (3), 791–819.