

Pre-Entry Test for EMTH118/MATH102 Solutions

Each completely correct question is worth one mark. If a question has two parts, each correct part is worth half a mark. The test is out of a total of 16 marks, and we would expect students enrolling in EMTH118 or MATH102 to get **at least** 75% or 12/16 correct.

1. If $x^2 + 8x + 5 = (x + a)^2 + b$, find the values of a and b . Answer: $a = 4, b = -11$.

2. Solve $\ln(2x + 1) - \ln(x - 1) = 2 \ln 4$. Answer: $x = \frac{17}{14}$.

3. If $f(x) = x^2 + 1$, give the equation of the line that passes through $(1, f(1))$ and $(3, f(3))$.
Answer: $y = 4x - 2$.

4. If $f(x) = x^3 - 2x$, find and simplify $f(x - 2)$. Answer: $f(x - 2) = x^3 - 6x^2 + 10x - 4$.

5. Solve $3 - p < p + 4$. Answer: $p > -\frac{1}{2}$.

6. Solve the following system of equations. Answer: $x = -2$ and $y = 1$.

$$2x + 3y = -1$$

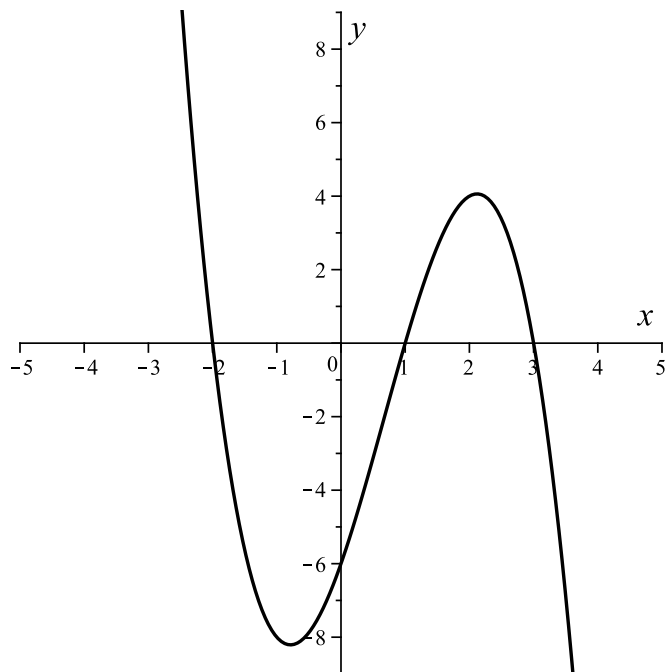
$$3x - 5y = -11$$

7. Find where the line $2x - y = 1$ intersects the circle $x^2 + y^2 = 2$.

Answer: $(1, 1)$ and $\left(-\frac{1}{5}, -\frac{7}{5}\right)$.

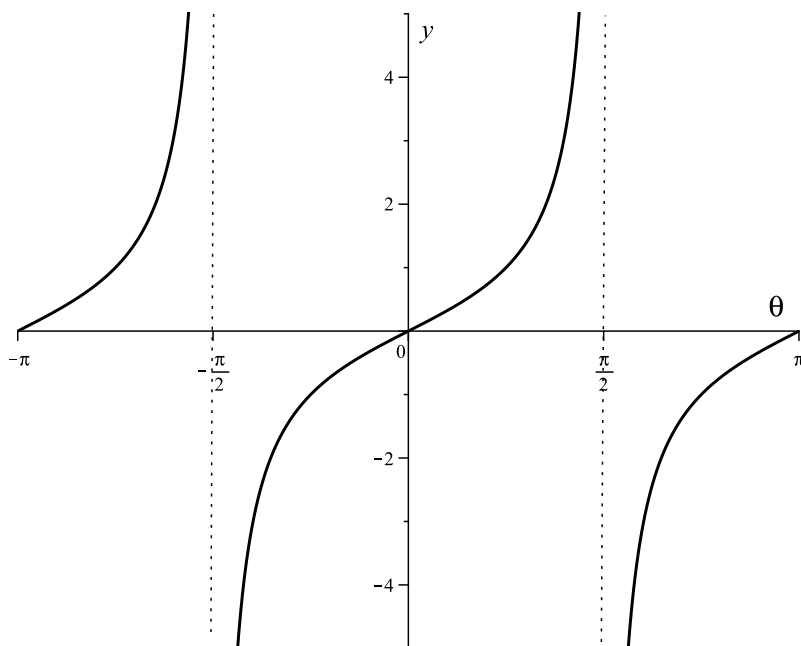
8. Sketch the graph of $y = (x - 1)(3 - x)(x + 2)$.

Note: To get the mark for this question, you should have labelled the x - and y - axes and given a suitable scale on each axis.



9. Sketch the graph of the trigonometric function $y = \tan \theta$ for $-\pi \leq \theta \leq \pi$.

Note that the graph should have labels on the x - and y - axes, a suitable scale on both axes, and show the vertical asymptotes at $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$.



10. Find the equation of the tangent line to the curve $y = 3x - x^2$ at the point $(4, -4)$.

Answer: $y = -5x + 16$.

11. Differentiate the function $y = e^{2x} - \frac{1}{x}$. Answer: $\frac{dy}{dx} = 2e^{2x} + \frac{1}{x^2}$

12. Find the derivative of the function $f(x) = x^2 \sin x$. Answer: $f'(x) = 2x \sin x + x^2 \cos x$.

13. Differentiate $f(x) = \sqrt{x^2 - 1}$. Answer: $\frac{x}{\sqrt{x^2 - 1}}$.

14. Evaluate $\int_0^3 (t^2 + 3t - 6) dt$. Answer: $\frac{9}{2}$.

15. Find the integral $\int \frac{2x}{\sqrt{x^2 + 4}} dx$ Answer: $2\sqrt{x^2 + 4} + C$.

16. Find the area under the curve $y = \sqrt{x}$ from $x = 1$ to $x = 4$. Answer: $\frac{14}{3}$.