Pre-entry Test for MATH103/EMTH119

This test is to help you determine if you have the background and skill needed to enrol in EMTH199 or MATH103. Print out the test, and work through the questions. The test should take **no more than one hour.** After completing the test, use the solutions file to mark your answers. (A marking guide is included in the solutions.) We would expect students enrolling in EMTH119 or MATH103 to get **at least** 75%, that is, **at least** 33/44.

1. If $f(x) = e^x$ and $g(x) = 5x + 1$, find

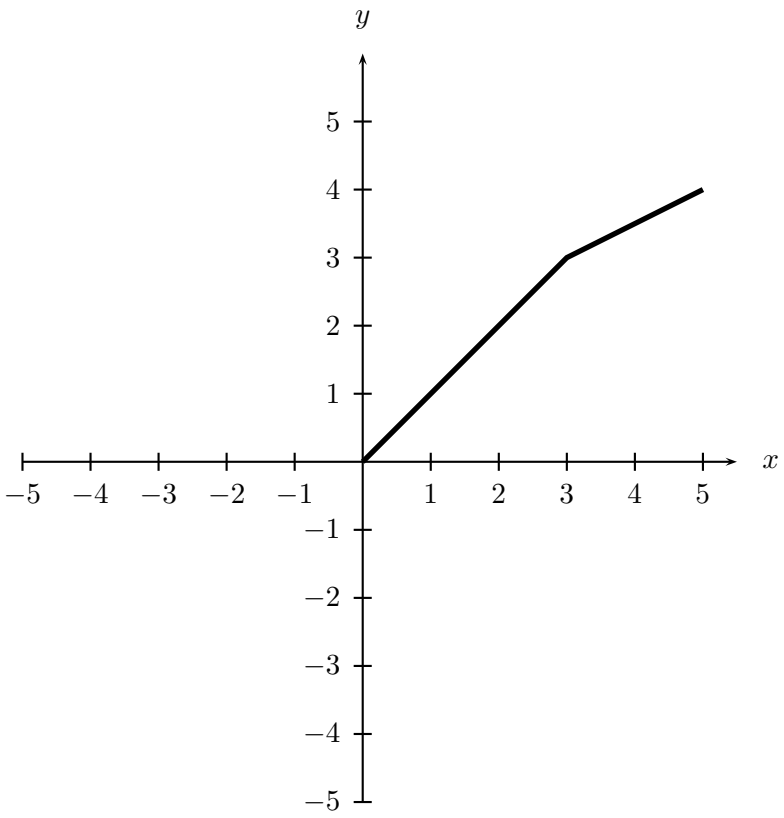
   (a) $g(x + 2)$

   (b) $g(x) + 2$

   (c) $f_\circ g(x)$

   (d) $g_\circ f(x)$

2. Let $y = f(x)$ be the function whose graph on the domain [0, 5] is shown below.



   (a) Extend the graph of f so that f is an even function on the domain $[-5, 5]$.

   (b) Extend the graph of f so that f is an odd function on the domain $[-5, 5]$.

3. Consider the function $f(x) = \sqrt{x + 4}$.

   (a) Show that f is one-to-one on $[-4, \infty)$.

   (b) Find a formula for f^{-1} on this domain.

   (c) What is the domain and range of f^{-1}?

4. Find the following limits, where possible. (If a limit does not exist, say so and use the symbols ∞ or $-\infty$ if appropriate.)

   (a) $\lim_{x\to 2^-} \dfrac{1}{x-2}$

   (b) $\lim_{x\to 3} \dfrac{x^2-9}{x-3}$

   (c) $\lim_{x\to\infty} \dfrac{x^3-2x+1}{2x^2-3}$

   (d) $\lim_{x\to\infty} \dfrac{3-x^2}{1+2x^2}$

5. Find the derivative of each of the following functions.

   (a) $f(x) = \cos(x^2-4)$

   (b) $y = \dfrac{1}{(1+x^3)^2}$

   (c) $g(z) = \ln(\cos z)$

   (d) $y = e^{x^3+1}$.

6. Use implicit differentiation to find $\dfrac{dy}{dx}$ where $\sin(x+y) = x$.

7. The equation

$$x^2 - xy + y^2 = 3$$

   represents an ellipse, whose axes of symmetry are not parallel to the coordinate axes.

   (a) Use implicit differentiation to find the slope of the tangent at a general point (a, b) on the ellipse.

   (b) Find the points where the tangent lines to the ellipse cuts the x-axis.

   (c) What do you notice about the tangent lines at these points?

8. Use a suitable substitution to find the following integrals.

   (a) $\displaystyle\int x\sqrt{1+x^2}\,dx$

   (b) $\displaystyle\int \frac{u}{u^2+7}\,du$

9. Use integration by parts to find:

   (a) $\displaystyle\int x^2\ln(x)\,dx$

   (b) $\displaystyle\int x\cos(x)\,dx$

10. Determine, with no or minimal calculation, whether the linear systems with the given augmented matrices have a unique solution, infinitely many solutions, or no solutions. Justify your answers.

(a) $\left[\begin{array}{cccc|c} 0 & 1 & 3 & \vdots & 1 \\ 1 & 0 & 1 & \vdots & 1 \\ 0 & 0 & 1 & \vdots & 2 \end{array}\right]$ (b) $\left[\begin{array}{cccc|c} 3 & -2 & 0 & \vdots & 1 \\ 1 & 2 & -3 & \vdots & -1 \\ 2 & 4 & -6 & \vdots & 0 \end{array}\right]$ (c) $\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & \vdots & 0 \\ 5 & 6 & 7 & 8 & \vdots & 0 \\ 9 & 10 & 11 & 12 & \vdots & 0 \end{array}\right]$

11. Consider the following matrices

$$A = \begin{bmatrix} 1 & -4 \\ -2 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 3 \\ 7 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Decide which of the following matrix expressions make sense, and evaluate those which do.

(a) $A - 3B$
(b) AC
(c) BD
(d) $BA - (DC)^T$

12. Decide, with reasons, which of the following matrices are invertible.

Find the inverse of those that are invertible.

(a) $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 2 & 3 & 5 \end{bmatrix}$ (b) $B = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & -1 \\ 0 & 1 & 0 \end{bmatrix}$

13. Consider the vectors $\mathbf{u} = (4, 2, 0)$ and $\mathbf{v} = (3, -1, 2)$.

(a) Find $\mathbf{u} \cdot \mathbf{v}$.
(b) Find $\mathbf{u} \times \mathbf{v}$.
(c) Find $||\mathbf{u}||$ and $||\mathbf{v}||$.
(d) Find the angle between $\mathbf{u}$ and $\mathbf{v}$.

14. (a) Find a vector parametric equation for the line passing through the points (1, −1, 2) and (3, −2, 1).

(b) Find a vector parametric equation for the line in 3-space which passes through the point (1, 0, −1) and is parallel to the line with the scalar parametric equations

$$x = 3 + t, \quad y = 4 \quad \text{and} \quad z = 2t.$$

(c) Write the line $x + 3y = 4$ in vector parametric form .

15. (a) Write down a normal of the plane $3x - y + 5z = -3$.

(b) Find a vector parametric equation for the plane passing through the points (1, 0, 0), (0, 1, 0) and (0, 0, 1).

(c) Write the plane in (b) in scalar form, $ax + by + cz = d$.