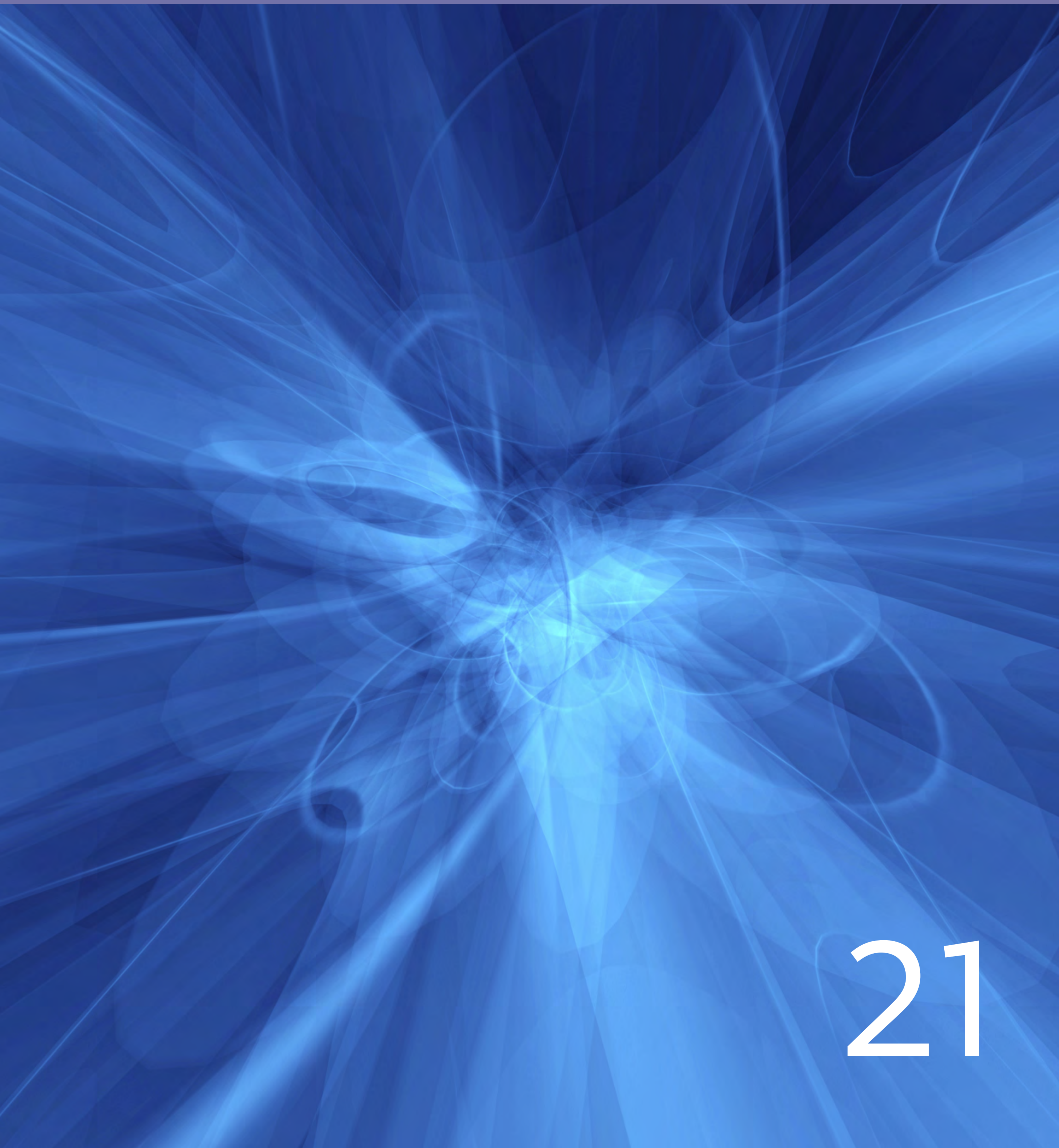
School of Mathematics and Statistics
College of Engineering

UNIVERSITY OF CANTERBURY
Te Whare Wānanga o Waitaha
CHRISTCHURCH NEW ZEALAND

Postgraduate Handbook 2021



School of Mathematics and Statistics

Postgraduate Programme 2021

Welcome from our School. Postgraduate study enables you to study selected mathematical and statistical topics in depth. There is a change of emphasis from the preceding undergraduate years: courses tend to be more focused on a specific problem or class of problems, rather than attempting to give a broad coverage of a branch of mathematics and statistics.

There is the possibility of taking one or more projects in which you investigate some problem with the assistance of a member of staff. Depending on the nature of the problem, this may involve literature searches, the use of various computing packages (for example `MATLAB`, `MAPLE` or `R`), resources on the internet, proving new theorems or data analysis. You will produce a written dissertation and may give an oral presentation.

Any proposed programme of study requires the approval of the 400 level coordinator. It is highly unlikely that any proposed programme that has a high workload in one semester will be approved, so you should try to construct a programme that balances your workload evenly over both semesters.

You can include some courses from other subjects (e.g. COSC480 is recommended for developing programming skills). This is a good way to ensure you have a broad program of study. Check with the School's 400 level coordinator that the courses from other subjects are suitable for inclusion within your program of study. In addition, there are a various joint programs between our School and other Departments/Schools detailed below.

At the Masters and PhD level you will undertake research, often focussed on deep study of a specialised topic. You will learn skills in undertaking systematic investigations, contextualising your work within the current state of understanding, so that your research outcomes can extend beyond the forefront of human knowledge.

Honours or PG Diploma?

The School offers both Honours and Postgraduate Diploma programs of study, which can be undertaken under Science or Arts. The most appropriate program is best decided on a case-by-case basis which you should discuss with the 400 level Coordinator, Prof. Michael Plank. The following guide provides some general advice about your options. You are welcome to get in touch as soon as possible, but you must do so before the formal enrolment period. If you are undertaking honours, you must have arranged a supervisor of your project in advance of enrolment. The 400 level coordinator can help you with this.

In addition, we also offer a Postgraduate

Certificate in Arts for students interested in a 60 points programme of study, or a Certificate of Proficiency for undertaking a course or courses of interest.

Who should think about Honours? If you view Mathematics/Statistics as more than a means to an end, then doing Honours will be a year well spent. In addition to taught courses, the honours program has a full year 30 point project which will not only deepen your understanding of a specialised topic but will also develop many of the soft skills desired by employers or for further Postgraduate study, like self-motivation, independent learning, research, written and oral communication.

The Honours subject majors are listed below. Formal details are in the UC Calendar. To enter Honours in Mathematics, you will need at least 60 points from MATH301-394, plus at least 30 points from 300-level MATH, STAT or other approved courses. For Honours in Statistics, you need at least 60 points of 300-level STAT301-394, plus at least 30 points from 300-level STAT, MATH or other approved courses. Normally you will have maintained at least a B+ average in these papers.

BSc (Hons) Major Subjects

In the Science faculty, Honours from our School may be completed in:

- Mathematics and/or Statistics, see Prof. Michael Plank;
- Data Science; see Prof. Jennifer Brown;
- Mathematical Physics, see Assoc. Prof. Jenni Adams (Physics);
- Computational and Applied Mathematical Sciences, see Prof. Michael Plank;
- Mathematics and Philosophy, see Prof. Clemency Montelle;
- Finance and Mathematics, see Assoc. Prof. Rua Murray;
- Finance and Statistics, see Assoc. Prof. Marco Reale; or
- Financial Engineering, see Assoc. Prof. Marco Reale.

These Honours programs require completion of papers totalling 90 points at 400 level or above (typically six 15 point courses) in addition to the 30 point MATH/STAT/CAMS449 project. In the case of data science, the project is 45 points.

BA (Hons) Major Subjects

In the Arts faculty, Honours from our School may be completed in Mathematics or Statistics. BA (Hons) consists of a project (MATH/STAT449) and six papers (120 points in total).

Who should think about a Postgraduate Diploma? The Postgraduate Diploma can consist entirely of taught courses, as there is no requirement that any project is undertaken. The entry requirements are as for Honours, except that you are not required to have a B+ average. It is very strongly recommended that your average grade in your majoring subject at stage 3 is at least a C+. The PGDipSc can also be used as Part I of a two part research MSc.

PGDipSci and PGDipArts

In both the Science and Arts faculty the Postgraduate Diploma can be taken in mathematics or statistics. In addition a Postgraduate Diploma in data science may be taken in the Science faculty. These diplomas require completion of papers totalling 120 points at 400-level or above (typically eight 15 point courses).

Masters in Applied Data Science

Data science is a new profession emerging along with the exponential growth in size, and availability of 'big data'. A data scientist provides insight into future trends from looking at past and current data. Data science is an essential skill in a world where everything from education to commerce, communication to transport, involves large scale data collection and digitalisation. New Zealand and other countries are currently experiencing a skills shortage in this area, and the need for data savvy professionals with applied experience is growing.

This 180 point conversion master's is designed to accommodate students from a range of undergraduate backgrounds (not just those with Mathematics, Statistics and Computer Science majors), who want to enhance or build their data science capabilities and combine these with the skills and knowledge they bring from their previous studies. So long as you are data-hungry and industry-aware; this degree can add to your employability and career prospects.

Masters in Financial Engineering

Financial engineering is a cross-disciplinary field combining financial theory, mathematics, statistics and computational tools to design and develop new financial or actuarial products, portfolios and markets. It also has an important role to play in the financial industry's regulatory framework. Financial engineers manage financial risk, identify market opportunities, design and value financial or actuarial (insurance) products, and optimize investment strategies.

The year long 180 point program consists:

- 135 points from taught courses. There is a core set of required courses in finance, mathematics & statistics and computer science. Further, there are a suite of suggested courses from these topic areas, that make up the majoring subject of Financial Engineering. Depending on your prior education, it is envisaged that around half of the taught courses will be MATH400 or STAT400 papers and the other half will be FINC600 papers; and
- the 45 point paper FENG601 Application of Financial Engineering which provides the opportunity to apply the techniques learned through the programme to real-world financial engineering problems.

There are minimum entry requirements into the program, which if not met you will be required to take FIEC601 in January-February prior to commencement of the program proper. You will be required to complete COSC480 Introduction to Programming, if you do not have equivalent programming skills (e.g. from COSC121, MATH170, EMTH171 or STAT221). Full details are provided in the UC Calendar.

Research Masters

A research Masters in Science (MSc) or Arts (MA) consists of two parts:

- Part I - a 120 points of papers (typically eight 15 point courses); and
- Part II - a 120 points research thesis.

Students can enter directly to Part II, if they have completed a Postgradute Diploma or Honours degree in the same majoring subject. For full details see the UC Calendar.

Our School offers the research MSc and MA in mathematics or statistics. An MSc is also offered in computational and mathematical sciences and data science. Enrolment in a Master's programme requires approval from the Postgraduate Coordinator, Dr Daniel Gerhard. At least one staff member must have agreed to supervise your Part II research study before approval of your programme of study.

PhD Research

The PhD programme is the highest degree offered in UC. How do you know if you are ready to purse a PhD in any of the following subjects we offer?

- mathematics;
- statistics;
- computational and applied mathematical sciences (CAMS);
- mathematical physics; and
- mathematics and philosophy.

The simplest answer is: if you are passionate about a subject and you want to get a deeper understanding of a field of study or want to use sophisticated tools from mathematical sciences to solve real world problems, then you are ready!

If you want to upscale your knowledge in the subject you love then a PhD in mathematics or statistics is the programme for you. On the other hand, if you have an interdisciplinary project in mind then a PhD in CAMS could be a good option for you.

Further details are available from the PG Office website, including scholarship information, here: `http://www.canterbury.ac.nz/postgraduate/phd-and-doctoral-study/` Excellent performance in a BSc (Hons) or BA (Hons) may provide sufficient training to undertake a PhD, thus obviating the need for a Masters degree. However, a PGDipSc or PGDipArts would not normally be sufficient.

400-600 LEVEL COURSES

The courses for 2021 are outlined on the CIS system `https://www.canterbury.ac.nz/courseinfo/GetCourseDetails.aspx`. In order to see all the offerings for, say, Mathematics, search for `MATH4`. The School reserves the right to cancel any course that has a low enrolment. This will be determined at the beginning of each semester.

It is also possible (and often desirable) to include courses from other subjects, see the Regulations in the Calender for details with each degree. Note that any STAT courses may be included in a Mathematics degree and vice versa. For multidisciplinary programmes like Financial Engineering and Data Science (which have courses across subjects) consult Schedule A of the BSc Hons regulation in the Calendar for a list of potential courses.

400-Level Projects

A broad range of possible projects are outlined below. However, this list is not exhaustive and other possibilities for projects are certainly possible. Project supervision is by mutual agreement of the supervisor and student. You should arrange your project by the end of the first week of term in 2021. It is suggested that you seek out possible supervisors before enrolment week.

You will hand in a written report on 6 October 2021 which will contribute 80% of the grade; the remaining 20% will be an oral presentation in Term 4.

Projects in Mathematics

Cryptographic Schemes and Diophantine equations
Felipe Voloch

A Diophantine equation is a polynomial equation in many variables where the coefficients and the solutions are restricted in some way, e.g., to be integers or rational numbers.

Solving a Diophantine equation is typically hard but it is easy to build an equation so as to have a predetermined solution. This is an example of a "one-way function" with potential applications to cryptography. It is essential for some of these applications that we know that the equation we build does not have additional solutions.

There are a number of choices to go with that for an honours project depending on the student's background and interest. One can look at the theoretical questions related to counting solutions to Diophantine equations or solving them. One can look at computational methods to solve these equations. Finally, one can look at the cryptography side of it, building or breaking or implementing these cryptosystems.

Designs and their code
Geertrui Van de Voorde

In this project, we will look at connections between design theory and coding theory. A famous example is provided by the (sporadic) Golay codes, which correspond to the Witt designs (and sporadic Mathieu groups). More generally, the support of the codewords in a perfect codes always form a t-design. This project can take multiple directions, according to your interests; including recent research on rank-metric equivalents of these classical links.

Axiomatic planes
Geertrui Van de Voorde

In the real (Euclidean) plane, we know that there is exactly one line through two different points and that there is exactly one line through a point that is parallel to a given line. Now these two properties can be taken as axioms and a new class of planes, called axiomatic affine planes, can be constructed. In particular, it is perfectly possible to construct such planes that have only a finite number of points and lines. Probably the most important conjecture in this area is the question what the possibilities are for the number of points in an axiomatic plane. This project can take multiple directions, according to your interests.

Generalised quadranges/polygons
Geertrui Van de Voorde

The incidence graph of a generalised quadrangle is characterized by being a connected, bipartite graph with diameter four and girth eight. We can think of them as a set of points and lines without triangles (hence the name). Generalised quadrangles have their own rich theory, which dates back to the work of Tits on groups of Lie type.

In this project, we will study finite generalised quadrangles: we look for constructions, classifications and characterisations. Generalised quadrangles fall into the more general class of generalised polygons (diameter n, girth $2n$). A classic theorem of Higman and Feit shows that a finite generalised polygon is either a di-gon ($n = 2$), a projective plane ($n = 3$), a generalised quadrangle ($n = 4$), a generalised hexagon ($n = 6$) or a generalised octagon ($n = 8$). This project studies generalised quadrangles and/or polygons and can take multiple directions, according to your interests.

Computations in group cohomology
Brendan Creutz

In this project you will study properties of the group $GL_2(\mathbb{Z})$ of invertible 2 x 2 matrices with integer coefficients and related groups obtained by reducing the entries modulo n. The main tool for this will be group cohomology, which sounds complicated but can be computed fairly easily by hand or using computer algebra software. In the project you will learn how to carry out such computations and then collect data about these groups with the hope of uncovering interesting patterns which you could then try to prove theoretically. The motivation for studying these particular groups comes from elliptic curves, and possible follow up projects at postgraduate level could explore this connection further. This project would be suitable for a student with basic knowledge of groups (as seen in MATH240) and finite fields or modular arithmetic (as seen for example in MATH220, MATH321 or MATH324).

Data driven analysis of dynamical systems
Rua Murray

Dynamical systems arise as solutions of differential equations, or in any situation where the state of a system updates iteratively with the passage of time steps (e.g., a descent algorithm for training a deep learning network). The local and global behaviour of dynamical systems is often determined by invariants of various kinds: fixed points, periodic orbits, invariant manifolds, invariant probability distributions. When the system is complex (due to very strong nonlinearities and/or high dimension), these objects are hard to find and analyse. In the last decade, a new family of tools has developed, loosely under the umbrella name of "Dynamic mode decomposition". These methods use samples from the dynamical system to build approximate transfer operators, from which eigenvectors can be extracted. The theory behind these methods remains undeveloped, there is a plethora of possible computational strategies, and any dynamical system can be analysed in this way. The emphasis in this project can be tailored to student interest.

Exploring Links between Topology and Combinatorics
Mike Steel

Topological methods turn out to have unexpected applications in discrete mathematics. One example is the use of the "Ham sandwich theorem" to show that two thieves can always divide up a necklace with k kinds of jewels using no more than k cuts. Another example is the link between the Möbius function of a partially ordered set and the Euler characteristic of an associated topological space. This project will suit a student who has taken MATH320 and is taking MATH428.

Integer Programming
Chris Price

This project looks at various applications of integer programming, and stochastic solution techniques. Applications such as set covering, packing, and partitioning will be looked at, as well as network interdiction problems. Variants of genetic algorithms will be looked at, and tested on a selection of these applications. MATH303 or similar required.

Maths Craft
Jeanette McLeod (Maths & Stats), Phil Wilson (Maths & Stats), David Pomeroy (Teacher Education)

Do you love mathematics and craft? Are you interested in the history, sociology, or psychology of mathematics? We are offering multiple interdisciplinary honours projects for keen students who answer yes to those questions. Maths Craft New Zealand is a non-profit public engagement initiative which makes mathematics accessible through craft. It was founded and is run by Drs Jeanette McLeod and Phil Wilson from the School of Mathematics & Statistics. Since Maths Craft's inception in 2016, we have run numerous festivals and workshops across New Zealand, and have reached over 11,000 people, making us the largest maths outreach programme in the country. Together with Dr David Pomeroy from the School of Teacher Education, we are studying the mathematical, historical, sociological, psychological, and pedagogical aspects of our craft-based approach to mathematics. If you like to think outside the box and can combine a love of mathematics and craft with a love of (or enthusiasm to learn about) social research, then please discuss project options with us.

Population dynamics: Bugs, beetles, plants, animals and diseases
Alex James

Populations, be they of bugs, plants or humans infected with viruses show a remarkable range of behaviours. Find out more about them in a project that uses dynamical system models to try and understand the dynamics of an example population. Examples systems include data driven dynamic models of New Zealand birds, the effect of initial viral load on COVID-19 mortality and assessing vaccination strategies for emerging diseases.

Lax Pairs
Mark Hickman

Given a non-linear differential equation, a Lax pair is a pair of linear differential operators $\mathcal{L},\ \mathcal{M}$ whose commutator vanishes only on solutions of the differential equation. A Lax pair allows one to potentially solve the differential equation by reducing the problem to an eigenvalue problem (if the operator $\mathcal{L}$ is second order, this is a Sturm-Liouville problem) and a time evolution of the eigenfunction; the so-called inverse scattering method. If $\mathcal{L}$ is first order then the Lax pair gives a conservation law of the differential equation. In this project, we will be looking at a method to compute the Lax pair of prescribed order for a differential equation (if it exists). This will involve Maple and would suit a student who has completed MATH302.

Combinatorics of reticulate evolution
Charles Semple

Evolution is not always a treelike process because of non-treelike (reticulate) events such as lateral gene transfer and hybridisation. In computational biology, the reticulate evolutionary history of a set of extant species is typically represented by a phylogenetic network, a particular type of rooted acyclic directed graph. Although reticulation has long been recognised in evolutionary biology, mathematical investigations into resolving questions concerning the combinatorial and algorithmic properties of phylogenetic networks are relatively new. Questions include, for example, how hard is it to decide if a given gene tree is embeddable in a given network? If we select a network with a certain number of leaves uniformly at random, how many reticulations do we expect it to have when the number of leaves is sufficiently large? When is a network determined by the path-length distances between its leaves? The aim of this project is to explore such questions. It involves discrete mathematics but there are no formal prerequistes.

Tutte's 5-Flow Conjecture
Charles Semple

Network flow problems are an important class of problems in combinatorial optimisation and represent a large variety of real-world occurrences. A particular type of network problem gives rise to Tutte's 5-flow conjecture (1954), amongst the most outstanding conjectures in modern-day graph theory. The purpose of this project is to investigate the progress that has been made on this conjecture and its connections to other areas of combinatorics. To whet your appetite, the Four Colour Theorem says that every planar graph without isthmuses has a 4-flow. While some prior knowledge of graph theory would be helpful, it is not a prerequisite for the project.

Topics in Group Theory
Gunter Steinke

Groups naturally occur as collections of symmetries of algebraic structures or geometries or algebraic structures or other objects. Knowing the structure of the group of symmetries of an object often leads to useful information about the underlying object. Groups come in very different sizes and forms and are also fascinating in their own right.

In one project we may look at abstract groups and how they can be seen as groups of symmetries. By making additional assumptions on transitivity we try to determine which groups can arise. For example, the symmetric and alternating groups are the only finite groups that are highly transitive, but there are many interesting groups that are 2-, 3- or 4-transitive.

In another project one may investigate so-called crystallographic groups. They arise as groups of symmetries of (perfect and unbounded) crystals. In two dimensions they are referred to as wallpaper groups. One looks at the underlying principles that allow to classify crystallographic groups and carry out a classification, for example, of wallpaper groups.

Dimension Theory
Gunter Steinke

While one has a precise notion of dimension for vector spaces, there often is an intuitive understanding of the dimension of a space (not necessarily a vector space) as the number of coordinates or parameters used to describe the space. However, this notion proved to be imprecise as discoveries in the early 20th century showed. There exist bijections between a line and a plane and also continuous maps from the unit interval onto the unit square. This led to the question of whether or not m-space $\mathbb{R}^m$ and n-space $\mathbb{R}^n$ can be *topologically* the same for different m and n. To answer this question various topological invariants have been devised.

Obviously, any useful invariant should assign n-space dimension n. While it is often easy to verify that n-space has dimension at most n, it is harder to establish equality.

The project investigates some possible definitions of the dimension of a (metric) space, their properties, when these dimensions agree and what examples of topological spaces there are for which they are different.

Projects in Statistics

Historical heights of army and navy recruits
Elena Moltchanova

The population distribution of human heights reflects prevailing environmental and sociological conditions. The historical records available, however, always present a biased picture. For example, only people "tall enough" were enlisted in the army, and the definition of "tall enough" varied with demand for soldiers. For the navy, the limitations also applied to people who were too tall to comfortably live on a ship. Using the information available from the historical army and navy records to recreate the population height distribution throughout the 17th-20th century thus presents some interesting challenges. Among many attempts taken to model such data, Bayesian methods are of particular interest due to their flexibility and ability to easily include temporal autocorrelation in the model. You will need to have taken a course in Bayesian inference and have solid knowledge of calculus (deriving conditional, marginal, etc. distributions) and good programming skills in either R, PYTHON or C/C++. (Other projects using Bayesian statistics are available, such as using reversible jump Markov chain Monte Carlo to model the Old Bailey's data or monitoring epi- demics and manufacturing processes.)

Projects in Data Science

Ethics of data science / Data science for ethics

Giulio Valentino Dalla Riva

Using a variety of mathematical, statistical, computational, ..., approaches, we are going to analyse from an hybrid ethical-technical point of view some fundamental data scientific algorithm. As an example, think about the "Recommended" videos on YouTube, the "watch next" movies on Netflix, the "Discovery" songs on Spotify, the "related coverage" news in the New York Times: what do they all have in common? They all suggest you, based on your history and characteristics, which bit of information to consider next. They define a user-dependent priority on the available information. They filter information for you, and they shape the way you see the world (or, at least, part of it).

Recommender systems are ubiquitous machine learning algorithms for prioritising information. Different technical decisions impact the fairness, openness, reliability, trust, and social benefit of them. We will investigate mathematical models and computational techniques to assess how state-of-the-art recommender systems perform and, eventually, propose alternative systems.

Familiarity with a scientific programming language (R, PYTHON, JULIA, . . .) is recommended.

Social network and online communities analysis

Giulio Valentino Dalla Riva

This is a open ended project. If you are interested in using and developing mathematical, statistical, data scientific tools and notions to analyse the behaviour of online communities in social networks, we can discuss it. We are probably going to use a variety of approaches: data wrangling, scraping, anonymisation, networks modelling, natural language processing, image analysis, ...The projects can have a varying degree of theoretical - applied content.

Familiarity with a scientific programming language (R, PYTHON, JULIA, . . .) is recommended. Knowledge of complex networks is welcome, but not strictly necessary (we can work around it). Original research projects in the area are encouraged.

Data Science Investigation of Historical Mathematical Tables

Giulio Valentino Dalla Riva & Clemency Montelle

The history of computational algorithms, numerical methods, and data analysis has long been under-studied in the history of mathematics. Indeed, historians have largely been put off by the sheer volume of evidence, the majority of which is in the form of numerical tables. From trigonometric functions, to instants of syzygies in the calendar, to subtle corrections of planetary positions, these tables of numerical data, sometimes containing thousands of data points, are the direct product of an historical author-scientist carrying out an algorithm with the explicit and implicit set of mathematical assumptions that characterises their scientific culture of practise.

Classical investigation of these tables has relied on the expertise of the historians in solving complex tasks such as identifying the relationships between tables ("was this table computed starting from this other table?"). In this project the student will try to develop neural network classifiers, generative models and other data science techniques to investigate the tables. Familiarity with a Scientific Programming language (R, Python, Julia, ...) is requested.

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